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Heat transfer in the recirculating region formed by a backward-facing step

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Abstract—Analysis of experimental data, obtained in the present work on the vortical structure of a recirculating flow formed by a backward-facing step, has allowed the development of an approximate calculation method for predicting velocity and temperature profiles, reattachment length and local friction and heat transfer coefficients. An analytical expression for heat transfer in the reattachment region is derived which correlates experimental heat transfer data for different types of flow with a sudden expansion.

1. INTRODUCTION

THE HYDRODYNAMIC and temperature fields of turbulence in a recirculating region formed by a backward-facing step have not been studied adequately enough for an understanding of the physical mechanism of heat transfer in the wall region of this flow, including the reattachment point where a substantial enhancement of heat transfer is noted by many authors. Moreover, empirical relations suggested by different authors for predicting heat transfer at the point of reattachment are found to be at variance with the existing experimental evidence. Thus, the heat transfer coefficients predicted in ref. [1] by the equation suggested in ref. [2] differ from those measured by 60–600%. An empirical relation suggested in ref. [3] leads to an error of 200% virtually within the entire range of Reynolds numbers investigated. Moreover, variations in the existing heat transfer data found in ref. [4] for the reattachment point make the problem of the prediction of heat transfer in this region as unsolvable within the scope of the existing integral theories.

Numerous attempts to employ the K–ε model of turbulence for predicting the above-mentioned flows have given satisfactory results only for flow regions far from the wall. Furthermore, the K–ε model overpredicts the local heat transfer coefficients at the reattachment point. The incorporation of new wall functions has not given impressive results.

2. EXPERIMENTAL FACILITY AND PROCEDURE

The experiments were carried out in a 500×300 mm subsonic wind tunnel one of the side walls of which was made up of two offset plates measuring $1000\times500\times30$ mm and forming a backward-facing step. The plates were maintained at a constant temperature.

Mean and fluctuating parameters of the hydrodynamic and temperature fields were measured by the hot-wire technique with the use of the DISA-55-M system. A specially developed method of linearized hot-wire signal separation was employed. For the hot-wire orientation as shown in Fig. 1 (where only hot wire A is depicted, and wire B being omitted, not to clutter up the figure), equations for calculating mean and fluctuating velocity components can be written in the final form as

$$v = \frac{1}{K_0K_2}\left(\frac{E_A}{B_A}-\frac{E_B}{B_B}\right)$$

$$u = \frac{1}{2K_0}\left(\frac{E_A}{B_A}-\frac{E_B}{B_B}\right)$$

$$\overline{v'^2} = \frac{1}{K_0^2K_2^2}\overline{\left(\frac{e_A}{B_A}-\frac{e_B}{B_B}\right)^2}$$

$$\overline{u'^2} = \frac{1}{4K_0^2}\overline{\left(\frac{e_A}{B_A}-\frac{e_B}{B_B}\right)^2}$$

$$\overline{u'v'} = \frac{1}{2K_0^2K_2^2}\overline{\left(\frac{e_A}{B_A}\right)^2}-\overline{\left(\frac{e_B}{B_B}\right)^2}$$

where

$$K_0 = (\sin^2\phi+a^2\cos^2\phi)^{1/2} = 0.742$$

$$K_2 = \frac{(a^2-1)\sin 2\phi}{\sin^2\phi+a^2\cos^2\phi} = 1.67$$

are the coefficients of hot-wire orientation; $\phi_A+\phi_B = 180^\circ$, $\phi_A = 136^\circ$ the inclination angles of wires A and B; $q^2 = q_n^2+a^2q_p^2$ ($a = 0.268$) the effective rate of hot-wire cooling; E_A and E_B the averaged output signals of the hot wires; e_A and e_B the fluctuating output signals of the hot wires; and B_A, B_B the

NOMENCLATURE

A, B	dimensionless constants
c_p	static pressure coefficient, $(P-P_s)/\rho u_\infty^2$
H	step height [m]
l	mixing length, $\sqrt{(\overline{u'v'})}/(\mathrm{d}u/\mathrm{d}y)$ [m]
M	Mach number
P	static pressure [Pa]
Pr	Prandtl number
q	velocity vector [m s^{-1}]
Re	Reynolds number
St	Stanton number
u, v	mean velocity components [m s^{-1}]
u', v'	fluctuating velocity components [m s^{-1}]
$\overline{u'v'}$	Reynolds stress [m^2 s^{-2}]
$\overline{v'\theta'}$	turbulent heat flux [m K s^{-1}]
x, y	Cartesian coordinates [m].

Greek symbols

δ	boundary layer thickness [m]
θ'	temperature fluctuation [K]
ν_t	turbulent viscosity [m^2 s^{-1}]
ρ	density [kg m^{-3}].

Subscripts

∞	free stream
R	reattachment point
s	separation point
w	wall.

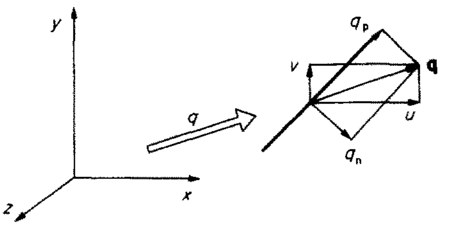

FIG. 1. Orientation of the probe hot wire A.

linearization (sensitivity) coefficients of the hot wires A and B.

For mean and fluctuating temperature measurements, an additional hot-wire probe (d = 2.5 μm) was employed which operated in the constant current regime. Its amplitude–frequency characteristic was adjusted automatically by a specially designed electronic corrector [5].

The technique described made it possible to measure turbulence parameters with the following maximum errors:

$$u = 4.15\text{–}9.15\%, \quad v = 8\%; \quad \sqrt{(\overline{u'^2})} = 5.15\%$$

$$\sqrt{(\overline{v'^2})} = 5.15\%; \quad \overline{u'v'} = 18.2\%; \quad \overline{v'\theta'} = 30.4\%.$$

Heat transfer and friction characteristics were determined from the slope of temperature and velocity profiles in the boundary layer viscous region. Basic measurements were made in flows past steps of two heights: H = 30 and 5.5 mm. The contouring of the opposite channel wall ensured a constant flow velocity of 16 m s^{-1} (13.3 m s^{-1}) with $Re_H = 3.38 \times 10^4$ (0.488×10^4) and the boundary layer thickness to step height ratio $\delta_s/H = 1.5$ (4.73). The values in parentheses correspond to the step of smaller height.

3. EXPERIMENTAL RESULTS

The vortical structure of the flow downstream of a backward-facing step was investigated in four sections of a recirculating region and in the region of flow relaxation after the point of reattachment. The effect of thermal boundary conditions on heat transfer was investigated with the heating of both the entire test section and a portion after the step.

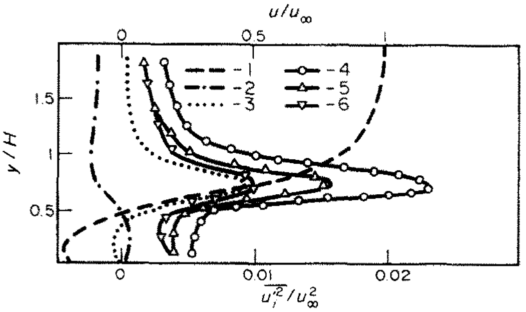

FIG. 2. Mean and fluctuating velocity components in the section $x/H = 2.5$: 1, u/u_∞; 2, v/u_∞; 3, $\overline{u'v'}/u_\infty^2$; 4, $\overline{u'^2}/u_\infty^2$; 5, $\overline{v'^2}/u_\infty^2$; 6, $\overline{w'^2}/u_\infty^2$.

The results of measurements of the mean and fluctuating turbulence parameters are presented in Figs. 2 and 3. The distribution of the integral flow characteristics is given in Fig. 4.

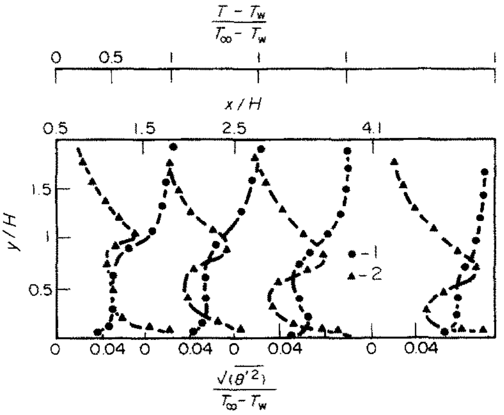

FIG. 3. Mean and fluctuating temperature components downstream of the step: 1, $(T-T_w)/(T_\infty-T_w)$; 2, $\sqrt{(\overline{\theta'^2})}/(T_\infty-T_w)$.

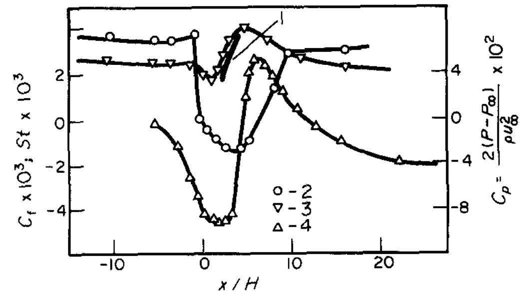

FIG. 4. Integral characteristics of the region studied: 1, $St_0 = 0.332 Re_x^{-0.5} Pr^{-2/3}$; 2, c_f; 3, St; 4, c_p.

The turbulent viscosity

$$\nu_t = \frac{\overline{u'v'}}{\mathrm{d}u/\mathrm{d}y}$$

and the mixing length

$$l = \frac{\sqrt{(\overline{u'v'})}}{\mathrm{d}u/\mathrm{d}y}$$

presented in Fig. 5, show that in the central region of the flow, between the separated boundary layer and the backward flow, there is a shear layer with nearly constant values of ν_t and l in cross-sections the magnitude of which increases along the flow proportionally with the longitudinal coordinate $l \approx 0.023x$.

3.1. *Flow in the vicinity of the wall*

The distributions of the mean and fluctuating characteristics in this zone are presented in Figs. 6 and 7. The observed substantial difference in velocity and temperature distribution from the 'wall law', despite an appreciable level of longitudinal velocity and temperature fluctuations, can be attributed to a predominantly laminar mode of flow in the boundary layer with relatively high external turbulence. The levels of the measured heat transfer and friction coefficients are more in line with the laminar boundary layer laws (Fig. 4).

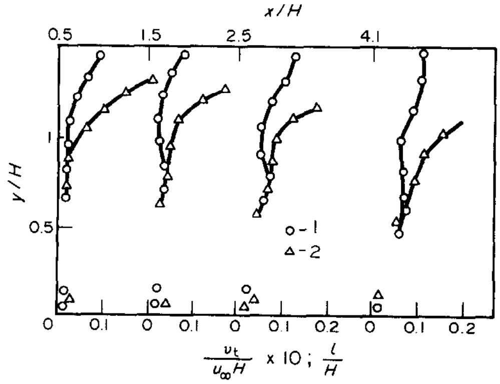

FIG. 5. Distribution of turbulent viscosity and mixing length in the mixing zone: 1, $(\nu_t/u_\infty H) \times 10$; 2, l/H.

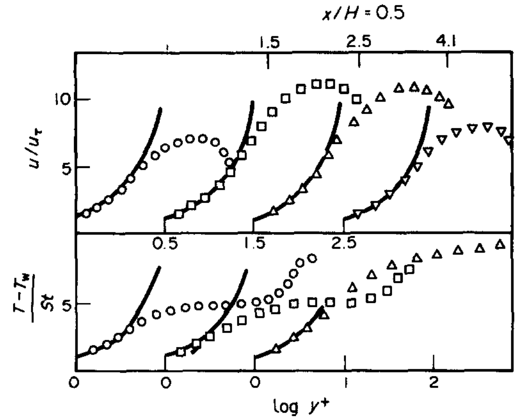

FIG. 6. Velocity and temperature profiles in the backward flow region in the wall coordinates.

4. PREDICTION OF FLOW CHARACTERISTICS

4.1. *Velocity and temperature profiles*

The velocity field was approximated with the aid of the 'Schlichting profile' [6]

$$\frac{u}{u_\infty} = \frac{\tilde{u}_1 + \tilde{u}_2}{2} + \frac{\tilde{u}_1 - \tilde{u}_2}{2} f(\eta)$$

where

$$f(\eta) = k(\eta - \eta^3) + \eta^3$$

$\tilde{u}_1$ is the velocity of straight flow over the boundary

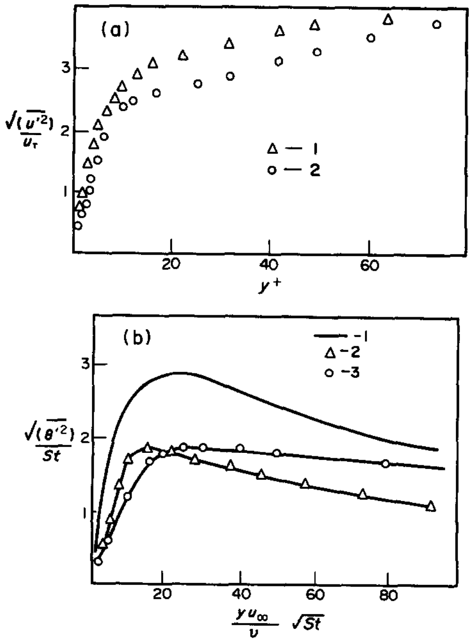

FIG. 7. Fluctuating velocity and temperature profiles in the backward flow region. (a) 1, $x/H = 1.5$; 2, $x/H = 2.5$. (b) 1, $\mathrm{d}P/\mathrm{d}x = 0$; 2, $x/H = 0.5$; 3, $x/H = 1.5$.

of the stretch, and $\tilde{u}_2$ the velocity of the backward flow over the boundary of the stretch.

The velocity field in the backward flow and in the separated boundary layer were approximated by the quadratic equations

$$\frac{u}{u_\infty} = A_0 + A_1 \frac{y}{b_*} + A_2 \left(\frac{y}{b_*}\right)^2$$

$$\frac{u}{u_\infty} = \tilde{u}_1 + E_1 \left(\xi - \frac{1}{2}\xi^2\right)$$

$$A_0 = -\frac{3}{4}\left[\frac{x}{x_R} - \left(\frac{x}{x_R}\right)^2\right]$$

where x_R is the length of the separation zone.

Coefficient A_2 is expressed so that the pressure distribution on the wall could be satisfied

$$A_2/b_*^2 = (30\tilde{x}^2 - 30\tilde{x}^4)/(\tilde{\delta}_s + 1).$$

The wall pressure coefficient is determined from the following relation:

$$\frac{c_p}{2} = \frac{P - P_s}{\rho u_\infty^2} = -\frac{A_0^2}{2} + 0.0013\left(\frac{x}{H}\right)^2 (10\tilde{x}^3 - 6\tilde{x}^5)/(\tilde{\delta}_s + 1)$$

where

$$\tilde{x} = x/x_R.$$

The equation for determining the separation zone length was obtained on the assumption of constant shear layer growth rate taking into account the curvature of the viscous flow upper boundary

$$(0.0168 - 0.023C)/(\tilde{\delta}_s + 1)x_R^2 + 0.13x_R - 1 = 0$$

where $C = 1 - (F_1/F_2)^2$, and F_1 and F_2 are the cross-sectional areas of the channel before and after the step, respectively. In Table 1 the predicted separation zone lengths are compared with experimental data obtained by other authors.

Table 1

Reference	Experimental x_R	Predicted x_R	Percentage error (%)
Present work	6.55	6.6	1
[7]	6.6	6.7	1
[7]	5.30	5.45	3
[8]	5–6	5.5	9
[9]	7	7.1	1.5
[10]	7.5	6.82	9.9
[11]	6	5.58	7.5
[12]	7.97–8.2	6.7	20
[13]	6	6.6	10
[14]	6	5.85	2.5
[9]	6	6.3	5
[9]	8	8.8	10

Temperature profiles given as $\overline{u\theta}$ were approximated analogously.

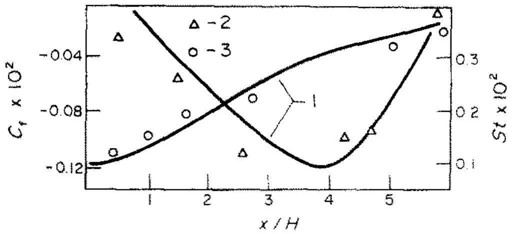

FIG. 8. Distribution of the friction factors and Stanton numbers in the backward flow region: 1, prediction; 2, c_f; 3, St.

4.2. *Friction and heat transfer coefficients*

These were determined from approximate solution for a laminar boundary layer developing from the reattachment point. The velocity profile was approximated by a cubic polynomial and temperature profile by a fourth-order polynomial. The solution is found taking into account the longitudinal pressure and temperature gradients under the conditions of the internal problem. The external turbulence was taken into account by the non-zero value of du/dy on the outer edge of the laminar boundary layer on the joining of the velocity and temperature profiles of the boundary layer and the wake.

Numerical solution of the integral equations of the laminar boundary layer has shown that the predicted local heat transfer and friction coefficients agree well with the experimental data obtained (Fig. 8).

The solution gave the following expression for the local Stanton number at the reattachment point:

$$St_R = 1.443\theta_{0R}\, Pr^{-2/3}\, Re_H^{-1/2}\, \tilde{x}^{-1/2}$$

where

$$\theta_{0R} = \frac{T_0 - T_{wR}}{T_\infty - T_{wR}}$$

T_{wR} is the wall temperature at the reattachment point, T_0 the temperature at the outer edge of the laminar boundary layer at the reattachment point.

The predicted maximum heat transfer rates at the reattachment point are compared with available experimental data in Fig. 9. Despite a certain difference among the results, it may be noted that the relation suggested correlates all of the available experimental data on heat transfer after a step, including those obtained for pipes with a sudden expansion. Moreover, the heat transfer data given in Fig. 9 were obtained over a wide range of flow velocities (from flows at 10 m s^{-1} to those with $M = 2.4$).

The relative heat transfer law at the reattachment point can be given in the form

$$\psi\, Pr^{-0.08} = 57.8 \frac{1}{(Re_\delta^s)^{0.25}(u_R/u_s)^{0.5}(x_R/\delta_s)^{0.5}}$$

where Re_δ^s is the Reynolds number based on the flow parameters at the reattachment point; u_R and u_s the potential flow velocities at the reattachment and sep-

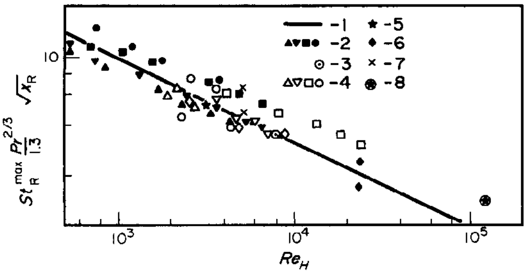

FIG. 9. Correlation of the data on heat transfer in the zone of flow reattachment: 1, $St_R^{max}(Pr^{2/3}/1.3)x_R = Re_H^{0.5}$; 2, ref. [2]; 3, ref. [16]; 4, refs. [14, 17]; 5, present paper; 6, ref. [15]; 7, ref. [3]; 8, ref. [18].

aration points, respectively; and δ_s the boundary layer thickness at the separation point.

The above expression shows that the presence in the flow of the backward-facing step may lead to both heat transfer enhancement ($\psi > 1$) and the formation of a thermal screen ($\psi < 1$) ast the point of flow reattachment.

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TRANSFERT THERMIQUE DANS LA REGION DE RECIRCULATION FORMEE PAR UNE MARCHE TOURNEE EN AVAL

Résumé—L'analyse de données expérimentales sur la structure tourbillonnaire de l'écoulement de recirculation formé par une marche tournée vers l'aval permet le développement d'une méthode de calcul approché des profils de vitesse et de température, de la longueur de réattachement et des coefficients locaux de frottement et de transfert thermiques. Une expression analytique pour le transfert de chaleur dans la région de réattachement unifie les données expérimentales pour différents types d'écoulement avec un brusque élargissement.

WÄRMEÜBERGANG IM RÜCKSTRÖMGEBIET EINER STUFENFÖRMIGEN QUERSCHNITTSERWEITERUNG

Zusammenfassung—Die experimentelle Untersuchung von Wirbelstrukturen an einer stufenförmigen Querschnittserweiterung erlaubt die Entwicklung eines Näherungsverfahrens zur Beschreibung von Geschwindigkeits- und Temperaturprofilen, der Länge des Ablösegebiets, der lokalen Reibung sowie der Wärmeübergangskoeffizienten. Für den Wärmeübergang im Wiederanlegebereich der Strömung wurde ein analytischer Ansatz gefunden, der aufgrund einer Korrelation von Versuchsergebnissen für verschiedene Strömungsarten an einer Querschnittserweiterung gewonnen wurde.

ТЕПЛООБМЕН В РЕЦИРКУЛЯЦИОННОЙ ОБЛАСТИ ТЕЧЕНИЯ ЗА УСТУПОМ

Аннотация—Анализ полученных экспериментальных данных по вихревой структуре в рециркуляционной области течения за обратным уступом позволил создать приближенный метод расчёта профилей скорости и температуры, длины участка присоединения и локальных коэффициентов трения и теплоотдачи. Выведено аналитическое выражение для теплообмена в зоне повторного присоединения, которое обобщает экспериментальные данные по теплоотдаче, полученные для различных типов внезапно расширяющихся потоков.